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The *Quadrivium* of 1008 and Pachymeres' *Syntagma*: Comparing two Byzantine *Quadrivia*

INTRODUCTION: THE TERM *QUADRIVIUM*

In the Middle Byzantine era the term *Quadrivium* is encountered in several *Vitae* (Lives) of saints, the very few examples of texts that are preserved from that period. The problems that are presented by the *Vitae* as historical sources are well known[1], while in the case that interests us the most essential fact is that a writer very often repeats or copies the previous ones. Another defect of the *Lives* of Saints is the fact that in many cases their authors didn't live in the period which they describe; therefore they are characterized by ignorance of both actual reality and terminology. In spite of these, the *Lives*, which were addressed to the wide public, expressed in a way the dominant ecclesiastical view, the commonly accepted, or rather the one considered to be commonly accepted. The common practice of the hagiographers was to refer to the education received by those whose lives they narrated, since education was considered an essential element for the character of a man, as important as parental care. The education is directly related to and defines for biographers the saint's childhood[2], a stage necessary to this literary genre. Furthermore, the detailed reference to the style and way of educa-

[1] P. Lemerle, *Le premier humanisme byzantin. Notes et remarques sur enseignement et culture à Byzance des origines au X^e siècle*, Paris 1971.

[2] A. Kiousopoulou, Χρόνος και ηλικίες στη βυζαντινή κοινωνία (= *Time and Ages in Byzantine Society*), Athens 1997, 71.

tion of the people that, their lives is narrated aims, frequently, at emphasizing the fact that even though they studied the classical sciences, they were not tempted by secular pleasures and didn't deviate from the path of God. However, the references to the classical sciences are quite illuminating, regarding their content and the method they were taught, even though this was not their purpose. A characteristic example is Ignatius the Deacon (born approximately in 780), *skeuophylax* of the church during the reign of emperor Theophilus, who in the *Vita* of patriarch Nikephorus[3] describes the studies of the emperor, mentioning the program of the mathematical quadrivium[4].

In the Middle Byzantine period, Leo the Mathematician, opposed probably to the veneration opinions, metropolitan of Thessalonica (842-843), seems to have taught mathematical sciences in Constantinople[5]. He was one of the most important intellectual related to the history of sciences in the Byzantine period during the reign of the emperor Theophilus, when he taught philosophy, but also arithmetic, geometry, astronomy and music, that is the fields of the *Quadrivium*. After Theophilus death, according to some historiographers, Caesar Vardas appointed Leo head of the philosophy school, which was founded after his initiative and his partly sponsorship at the School of Magnaura Palace, inside the Mega Palation, with a programme based on the form of the *Quadrivium*[6]. This was the first official school of higher education in the middle Byzantine period. The important fact here is the testimony that in Constantinople in the second half of the 9th century all those students who wished to become part of state bureaucracy were necessarily taught the fields of the *Quadrivium*, while the emperor himself showed great interest in this matter.

[3] Βίος τοῦ ἐν ἁγίοις πατρὸς ἡμῶν Νικηφόρου, *Nicephori archiepiscopi Constantinopolitani Opuscula Historica, Accedit Ignatii Diaconi Vita Nicephori*, Lipsiae 1880.

[4] Βίος τοῦ ἐν ἁγίοις πατρὸς ἡμῶν Νικηφόρου (cit. n. 3), 150-151.

[5] Lemerle, *Le premier humanisme* (cit. n. 1), ch. 6.

[6] *Theophanes Continuatus*, ed. I. Bekker, Bonn 1838, 192.

The study of the *Quadrivium* from the 9th century onwards seems to continue uninterruptedly. From the period of Constantine VII Porphyrogenitus, when particular emphasis was given on education and its organization, the study of the four mathematical sciences became an integral part of the educational curriculum. Of course this study aimed at the provision of general education as part of the educational 'program', which was a necessary prerequisite for anyone wishing to follow a career in state offices. In that period, during the reign of the emperors of the Macedonian dynasty, it seems that the form of the *Quadrivium* became consolidated as a complete and special part of the educational «program», and this continued until the fall of Constantinople in 1453. In the period of the Macedonian dynasty, in 1008, during the reign of Basil II, the oldest complete handbook for teaching the fields of the *Quadrivium* of the Byzantine period was written, which was preserved in many copies (some of these copies under the name of Michael Psellus, as the first editions in Venice and Paris[7], a mistake which was corrected by Valentin Rose in 19th century[8]).

An important mention of the term quadriviun, in the Greek version τετρακτύς, give us Anna Komnena (1083-1153/1154), who in her work *Alexiad* (Ἀλεξιάς) describing the content of her educational experience, refers to the lessons of the τετρακτύς[9].

[7] First edition in Venice: Τοῦ σοφωτάτου Ψελλοῦ, σύνταγμα εὐσύνοπτον εἰς τὰς τέσσαρας μαθηματικὰς ἐπιστήμας, Ἀριθμητικήν, Μουσικήν, Γεωμετρίαν καὶ Ἀστρονομίαν (*Doctissimi Pselli opus dilucidum in quattuor Mathematicas disciplinas, Arihmeticam, Musicam, Geometriam & Astronomiam*), Venetiis 1532. First edition in Paris: Τοῦ σοφωτάτου Ψελλοῦ, σύνταγμα εὐσύνοπτον εἰς τὰς τέσσαρας μαθηματικὰς ἐπιστήμας, Ἀριθμητικήν, Μουσικήν, Γεωμετρίαν καὶ Ἀστρονομίαν (*Doctissimi Pselli opus dilucidum in quattuor Mathematicas disciplinas, Arihmeticam, Musicam, Geometriam & Astronomiam*), Parisiis, Excudebat Iacobus Bogardus, 1545.

[8] V. Rose, *Pseudo-Psellus und Gregorius Monachus*, «Hermes», 2/3 (1867), 465-467.

[9] Πορφύρας τιθήνημά τε καὶ γέννημα γραμμάτων οὐκ ἄμοιρος, ἀλλὰ καὶ τὸ ἑλληνίζειν ἐς ἄκρον ἐσπουδακυῖα καὶ ῥητορικῆς οὐκ ἀμελετήτως ἔχουσα καὶ τὰς Ἀριστοτελικὰς τέχνας εὖ ἀναλεξαμένη καὶ τοὺς Πλατωνικοὺς διαλόγους καὶ τὸν νοῦν ἀπὸ τῆς τετρακτύος τῶν μαθημάτων πυκάσασα, *Comnenae Annae, Alexiadis*, ed. and tr. by L. Schopen, Bonn 1837, 1, 2.

It seems that the term τετρακτύς was widely used amongst the educated Byzantines, until the Paleologean Period. For instance, Nicephorus Gregoras (1295-1360), an important historian and astronomer, uses the term *Quadrivium* to indicate an educational program about the mathematical sciences, something we find in various works with the same content but entitled differently. Equally, George Pachymeres (1242-1310), teacher in the Patriarchate School, wrote in around 1300 a handbook of the four mathematical fields, and named it Τετράβιβλος or *Syntagma of the four lessons, arithmetic, music, geometry and astronomy*, only the second handbook from Byzantium to be preserved[10].

In this paper we are going to comparing the two available handbooks (the one of the year 1008 and the *Syntagma* of Pachymeres), to trace their differences and similarities.

THE TWO TEXTS

The quadrivium entitled Εὐσύνοπτον σύνταγμα εἰς τὰς τέσσαρας επιστήμες, the oldest teaching handbook of the four sciences preserved in its entirety from the Byzantine times[11], is written in 1008. This era was preceded by intense interest both in education as an institution on the part of imperial authority and by the revival of the discussion about mathematical sciences, particularly among the group of Byzantine scholars. So, it shows an effort for the creation of a teaching manual in the form of τετρακτύς[12], in order to satisfy the educational needs of all those continuing their studies on a higher level, after the first one (ἱερὰ γράμματα) and the level of *Trivium* (rhetoric, grammar and logic).

On the other hand, the *Quadrivium* entitled Σύνταγμα by George Pachymeres was written in around 1300, a period which is

[10] P. Tannery, *Quadrivium de Georges Pachymère*, Citta del Vaticano 1940.

[11] It was edited by J. L. Heiberg under the title *Anonymi, Logica et Quadrivium, cum Scholiis Antiquis*, Kopenhagen 1929.

[12] For the Byzantine curriculum of τετρακτύς, see G. Katsiampoura, *Πρόσληψη, μετάδοση και λειτουργία των επιστημών στους μεσοβυζαντινούς χρόνους και το* Quadrivium

commonly called the 'Paleologean Renaissance', characterized by the retrieval of the largest part of ancient knowledge and an introduction of the Arabic elaborations. The examination of its content reveals the high level of the study of sciences in the centuries that followed from the previous handbook.

The comparative study of the two handbooks allows us to discern what Byzantines considered educational needs, so as to form an idea of the cultural capital, as Pierre Bourdieu writes[13], of the specific social formation in the two different periods. In addition, it inform us what was a legitimate content of sciences, or – to use the wellknown term coined by the American historian of science Thomas Kuhn[14] – which was the dominant scientific Paradigm, as through the ideological mechanism of education, in the social formation of Byzantium, as well as which was the educational methodology favored during the two periods.

CONTENT

The Εὐσύνοπτον σύνταγμα *of 1008*

In the beginning of the 11th century, judging from the manuscripts preserved as well as their content and their number, most of the scientific works that have been written until the period of Late Antiquity were known, in the worst case even in contemporary commentaries. The Εὐσύνοπτον σύνταγμα is an epitome of these works, while frequent references to ancient writers which obviously aim at strengthening its credibility and validity. The function of the handbook is the reason for its obvious didactic character and its simplistic nature. The structure of the text reveals that the role of

του 1008 (= *Perception, Transmission and Function of Science in Middle Byzantine Era and the* Quadrivium *of 1008*), Ph.D. Dissertation, Department of Sociology, Panteion University of Social and Political Science, Athens 2004.

[13] P. Bourdieu, *The Forms of Capital*, in J. Richardson [ed. by], *Handbook of Theory and Research for the Sociology of Education*, New York 1986, 241-258.

[14] Th. Kuhn, *The Structure of Scientific Revolutions*, Chicago 1962.

the Εὐσύνοπτον σύνταγμα was an introductory one and functioned in combination with the main works of the ancient scientists which were used as its foundation, that is Nicomachus of Gerasa, Euclid, and Claudius Ptolemy.

The text refers repeatedly to Euclid and Ptolemy, it mentions Archimedes, Pythagoras and Plato, while, without mentioning him by name, it has been influenced greatly by the *Arithmetica* (Ἀριθμητικά) of Nicomachus of Gerasa. The direct reference to the ancient writers probably signify that it added more status and credibility to the text and that the students should obtain at least knowledge of their names. The frequent reference to Plato might be related to the later study of his work, which appeared as the main ideological trend in Constantinople after the middle of the 11th century, with main representative Michael Psellus.

The text follows a specific structure, which is specialized in every unit: definitions, essential principles, ways of calculation, a conclusion about the value of science.

Starting with arithmetic, the writer explains his choice by the value he attributes to this science and its dominant role compared to the other ones[15]. This view, which is expressed figuratively, refers directly to the view on arithmetic by Nicomachus of Gerasa, as expressed in *Arithmetica*[16].

The direct reference to the Nicomachean evaluation of sciences is yet another indication of the influence of Nicomachus, obvious

[15] Αὕτη καθηγεμὼν καὶ πρωταίτιος ἅτε καὶ ἁπλουστέρα καὶ στοιχειώδης καὶ πρὸς ἐκείνας [the other sciences] εὐθέως διαβιβάζουσα, *Anonymi, Logica et Quadrivium* (cit. n. 11), 51.

[16] Τίνα οὖν ἀναγκαῖον πρωτίστην τῶν τεσσάρων τούτων μεθόδων ἐκμανθάνειν; ἢ δηλονότι τὴν φύσει πασῶν προϋπάρχουσαν καὶ κυριωτέραν ἀρχῆς τε καὶ ῥίζης καὶ οἱονεὶ πρὸς τὰς ἄλλας μητρὸς λόγον ἐπέχουσαν. Ἔστι δὴ αὕτη ἡ ἀριθμητική οὐ μόνον, ὅτι ἔφαμεν αὐτὴν ἐν τῇ τοῦ τεχνίτου θεοῦ διανοίᾳ προϋποστῆναι τῶν ἄλλων ὡσανεὶ λόγον τινὰ κοσμικὸν καὶ παραδειγματικόν, πρὸς ὃν ἀπερειδόμενος ὁ τῶν ὅλων δημιουργὸς ὡς πρὸς προκέντημά τι καὶ ἀρχέτυπον παράδειγμα τὰ ἐκ τῆς ὕλης ἀποτελέσματα κοσμεῖ καὶ τοῦ οἰκείου τέλους τυγχάνειν ποιεῖ, ἀλλὰ καὶ ὅτι φύσει προγενεστέρα ὑπάρχει, ὅσω συναναιρεῖ μὲν ἑαυτὴ τὰ λοιπά, οὐ συναναιρεῖται δὲ ἐκείνοις, *Nicomachi Geraseni Pythagorei introductionis arithmeticae libri II*, ed. P. Hoche, Lipsiae 1866, 1, 4.

by the definitions of the numbers, that are directly influenced by the mystical Pythagorean perception supported by Nicomachus, as well as by the references to Pythagoras.

The initial definitions given in the handbook try to combine the ancient Greek perception regarding the numbers with the Christian world view. Therefore, number one, for example, is defined with its similarity to the divine[17].

The structure of the material tries to fulfill the educational objectives for which it was written. So, in order to facilitate the understanding of the content, in every unit we have first the definition and then a more specialized presentation, while then examples follow for better comprehension.

The fact that the handbook was addressed to the students of the higher stage of Byzantine education is evident by the content's level of difficulty. The anonymous author considers that the readers have already some knowledges, at least of an elementary level, that they have been taught essential arithmetic and calculations. So he makes the presentation of more special issues, that is he takes care to examine them deeply, although to a limited extend. In the Εὐσύνοπτον σύνταγμα we can detect an effort to analyze further, since particular care is given to the presentation of both research methods and methods of construction.

The conclusion of the unit is interesting since the writer presents his view on sciences. Escaping from the narrow 'scientific' framework in which he was limited so far, he makes an effort to present briefly some of the principles of his world view. The presentation of essential principles and views is traditionally one of the conventions of the writers of scientific works. Such views are presented by Claudius Ptolemy, for example, and as a matter of fact rather extended, in the preamble of *Syntaxis mathematica* (Μαθηματική σύνταξις)[18]. In the

[17] Ἡ μονὰς ἀριθμὸς οὐκ οὖσα γεννητική ἐστιν ἀριθμῶν πηγὴ καὶ ῥίζα καὶ ἀφορμὴ πλήθους παντὸς εἰκόνα σώζουσα θείου, ὃ μηδὲν ὂν τῶν ὄντων, ἔστιν δ' οὖν ὅμως τῶν ὄντων ποιητικόν, *Anonymi, Logica et Quadrivium* (cit. n. 11), 51.

[18] Ptolemaeus, *Syntaxis mathematica*, I, ed. J. L. Heiberg, Lipsiae 1898, Προοίμιον.

text under examination, the unknown author, on the occasion of the examination of the sphere, finds the opportunity to express his view on science and its relationship with the divine and the eternal. His view seems to be influenced by Plato, when he speaks about ἀπομόργματα (matrix) and the relationship between the eternal truths and man's ability to perceive them.

The unit about geometry starts with the same style of presentation that is stating the definitions of what are considered the basis for further study. The opening definitions have to do with the essential principles of geometry, for example the point (σημεῖον), the line (εὐθεία), etc. This is consistent with the curriculum in Byzantium, and therefore with the educational objective this handbook was written for. The students, according to the educational 'programme', have never before encountered the principles of geometry, so their teaching should commence from the beginning.

The presentation follows, partly, the one followed by Euclid in his *Elements*. The 1st Book of the *Elements* starts with a series of twenty-three definitions[19]. The definitions of the point, the line and the surface come first. The anonymous writer does not follow exactly the ancient mathematician, but he tries to use as simple definitions as possible.

Then, while the *Elements* provide all the definitions initially, the writer of the Εὐσύνοπτον σύνταγμα analyzes every issue that is going to be analyzed. Obviously he believes that this will respond better to the educational function of the handbook. The content of the unit on geometry is based mainly on the 1st Book of the *Elements*.

It is worthmentioning the fact of the multiple references to Euclid's *Elements*. Frequently, he does not follow the demonstration of proof to their end, but refers directly to the *Elements* for a more in depth analysis (for example, when he refers to the doubling of lines, he writes «like the findings of the 19th chapter of the 6th ele-

[19] See E. S. Stamatis, *Ευκλείδου Γεωμετρία. Στοιχεία Βιβλία 1, 2, 3, 4* (= *Euclid's Geometry. Elements, Books 1, 2, 3, 4*), Athens 1975.

ment of Euclid»[20]). The frequent reference of the *Elements* and the continuous quotations from it means that the ancient text was still in use and, moreover, it was an easy to use tool, at least to teachers[21]. This confirms the popularity of the Euclidian text, whose first complete copy is the one of 888[22]. Therefore, it is possible that the teaching was combined, that the *Elements* were used when someone wished to delve deeper into the issues set by the Εὐσύνοπτον σύνταγμα, that's why, as mentioned before, the content extends mainly in the first book, the introduction to the work of Euclid.

Worth pointing out is, also, that in this unit we meet continuous references to ancient philosophers, obviously because their authority is unquestionable. This is true even more in the case that it has been preceded by an extended reference to the mystical cosmology of Plato, which refers to the dialogue *Timaeus*[23]. The unknown Byzantine writer of the Εὐσύνοπτον σύνταγμα, after he mentioned the primary role of the triangles, refers to Plato in order to emphasize his point: ταύτη τοι καὶ Πλάτων μυστικὸν τινὰ λόγον τῶ τριγώνω ἐναποκρύπτεσθαι ἔλεγεν οἷα πανταίτιον καὶ ποιητικὸν τῶν σχημάτων ἁπάντων ἀναφαινόμενον.

What is mentioned above could be related here with all those mentioned by the writer and reveal his apparent personal mystical perception of the world. This is obvious when he refers, for

[20] *Anonymi, Logica et Quadrivium* (cit. n. 11), 90.

[21] The unknown writer quotes the *Elements* also to increase the prestige of his words and when he considers that this ancient work is obscure he tries to make it more explicit. He writes, for example: ὡς ἐν στοιχείῳ πρώτῳ λε΄ Εὐκλείδου κεφάλαιον, ὃ καὶ ἡμεῖς ἐπὶ τὸ σαφέστερον διαγράψομεν, *Anonymi, Logica et Quadrivium* (cit. n. 11), 81.

[22] See Katsiampoura, *Perception, Transmission* (cit. n. 12), 140.

[23] In his work *Timaeus* Plato writes: Πρῶτον μὲν δὴ πῦρ καὶ γῆ καὶ ὕδωρ καὶ ἀὴρ ὅτι σώματά ἐστι, δῆλόν που καὶ παντί· τὸ δὲ τοῦ σώματος εἶδος πᾶν καὶ βάθος ἔχει. Τὸ δὲ βάθος αὖ πᾶσα ἀνάγκη τὴν ἐπίπεδον περιειληφέναι φύσιν· ἡ δὲ ὀρθὴ τῆς ἐπιπέδου βάσεως ἐκ τριγώνων συνέστηκεν. Τὰ δὲ τρίγωνα πάντα ἐκ δυοῖν ἄρχεται τριγώνοιν, μίαν μὲν ὀρθὴν ἔχοντος ἑκατέρου γωνίαν, τὰς δὲ ὀξείας· ὧν τὸ μὲν ἕτερον ἑκατέρωθεν ἔχει μέρος γωνίας ὀρθῆς πλευραῖς ἴσαις διῃρημένης, τὸ δ' ἕτερον ἀνίσοις ἄνισα μέρη νενεμημένης. Ταύτην δὴ πυρὸς ἀρχὴν καὶ τῶν ἄλλων σωμάτων ὑποτιθέμεθα κατὰ τὸν μετ' ἀνάγκης εἰκότα λόγον πορευόμενον· τὰς δ' ἔτι τούτων ἀρχὰς ἄνωθεν θεὸς οἶδεν καὶ ἀνδρὸς ὃς ἂν ἐκείνω φίλος ᾖ, Plato, *Τίμαιος*, ed. V. Kalfas, Athens 1995, 53c-d.

example, to analogies, where he believes that «poetical wisdom», the wisdom of the Creator can be discerned: ἡ τοῖς ποιήμασιν ἐνθεωρουμένη ποιητικὴ σοφία κατὰ τὸ ἐγχωροῦν ἀνθρώποις διαγινώσκεται[24].

Plato mentions another case. And here it is noteworthy the framework in which it is mentioned, that is the explicit reference to the religion of the twelve gods. The Byzantine writer, mentioning the doubling of the cube, quotes the story with the oracle of Apollo to the Athenians and how Plato solved the problem and saved his fellow citizens from the plague[25]. The quotation of the story takes place with no attempt to scorn the ancient religion and in style that does not question the truth of the incident. Here we should mention that in an era when the image of the world was changed, it is impressive that whatever is related to a discourse on science is not questioned, at least regarding the specific social category, that of scholars, in which obviously the unknown writer is also part of.

Archimedes is also part of the group of scientists mentioned in the work. In an attempt to make easier to understandthe use of *dioptra* (a sighting tube) and to emphasize its value, the writer quotes its use by Archimedes, who applied it to measure the height of the pyramid. Finally, among the above mentioned scientists Theon of Alexandria, a commentator of Ptolemy, is also included as well as Ptolemy himself. The references to the ancient scientists and o Antiquity seem not to be absolutely compatible with the writer's world view, as he expresses it in the last paragraph of the unit. After he mentioned everything he considered adequate for geometry, he ended up with the conclusion that someome can not see God through science alone, but he also need virtue, obviously Christian virtue and faith. Because science by itself is not enough, by itself mathematical knowledge can be wrong, while virtue can exist by itself, he explains[26].

[24] *Anonymi, Logica et Quadrivium* (cit. n. 11), 99.

[25] *Anonymi, Logica et Quadrivium* (cit. n. 11), 92.

[26] Ὡς ἀρετῆς ἄνευ μαθηματικὴν τοῦ παντὸς ἁμαρτάνειν, μαθηματικῆς δὲ χωρὶς τὴν ἀρετὴν καὶ καθ' αὑτὴν τοῦ παντὸς εὐμοιρεῖν, *Anonymi, Logica et Quadrivium* (cit. n. 11), 103.

We could say that the conclusion of the unit is in a way a declaration of faith by the unknown writer, who considers necessary to include some Christian principles in a text that is based exclusively in the Pagan heritage. And although it may seem strange to present day readers, it should probably be considered as additional example of Byzantine eclectic perception.

The unit of astronomy starts similarity with no introductory observations or evaluations, moving gradually to more difficult elaboration. Therefore, the primary definitions have to do with the sphere, its centre and diameter, while essential is considered the geocentric system and its fundamental principles are described. From there he moves to the primary principles of movement of the system: the point in which he focuses has to do with the movement and the alternating locations of the five planets, the sun and the moon. Referring to the principles that govern the movement of the moon, he makes a rather dismissive criticism to the «old ones», because he believes that they did not analyze sufficiently several points relating its movement in the epicycle and because it differs regarding its apparent movement compared to the rest of the planets[27]. At this point we see that an effort is being made to evolve the existent data with further study. In this specific case the non apparent movement of the moon is interpreted with the combination of movements that result in the shaping of the final image of the movement forward.

The rules of the movement and the occupation of alternating locations by the planets, the sun and the moon, are followed by the interpretation of the phenomenon of the eclipse. Eclipses played a significant role in the scientific debate carried out in Byzantium, both as strange phenomena that required a natural explanation and regarding their prediction which came to constitute the philosophers' stone for the recognition of some as scientists. Famous is the case of the debate between Nicephorus Gregoras and Barlaam of Calabria, in the

[27] Ὁ γὰρ περὶ τούτου λόγος, οὐκ οἶδ' ὅπως, τοῖς παλαιοῖς σεσιώπηται, *Anonymi, Logica et Quadrivium* (cit. n. 11), 106.

14th century[28]. In the work under examination as well, a particular importance is given to the interpretation of this phenomenon.

The role played by Euclid's *Elements* in the unit on geometry is also played in the present unit on astronomy by the *Handy Tables* (Πρόχειροι κανόνες) of Ptolemy. Regarding the calculations, the reference is made to the respective stele of the tables of Ptolemy, something signifying that the *Syntaxis Mathematica* and the *Handy Tables* were easy to be found and studied. An important effort, and absolutely necessary, made by the unknown writer is demonstrating the contemporary character of the work of Ptolemy and its incorporation in the Byzantine system of calculating time. So a detailed presentation of the Egyptian calculation of time is presented, used by Ptolemy, so that the Byzantine reader may have the opportunity to convert it.

The last part of this unit possesses a completely different character. It is dedicated to the presentation of the constitution of the celestial bodies, and their influence to terrestrial phenomena. Therefore, there is a list of the qualities and their combinations that shape the character of its celestial body, as well as their influences due both to their qualities and their combination with the various locations on the zodiac. This last part, which is not directly related to the mathematical astronomy of the previous part, could also appear strange to the present day reader. However, it is another proof of the close relation between astronomy and astrology, in the present day meaning of the word, in the period of Late Antiquity and the Byzantine period. Following this, and obviously understanding that the last part of his treatise carries the danger of misinterpretation regarding the possibility and justification of predictions, the writer does not fail to express his opposition to all those who believe that they can make an evaluation of these developments[29].

[28] About this debate, see G. Katsiampoura, Νικηφόρος Γρηγοράς εναντίον Βαρλαάμ Καλαβρού: μια πολιτική διαμάχη με ένδυμα την πρόβλεψη των εκλείψεων (= *Nikephoros Gregoras against Barlaam of Calabria: A Political Argument as a Debate about Eclipses*), «Neusis», 13 (2004), 138-148.

[29] Ἀλλὰ σκαιοὺς καὶ ἀμαθεῖς λογιστέον τοὺς τὰ τοιαῦτα λέγειν ἐπιχειροῦντας, οἳ πορισμὸν βίου τὴν τερατολογίαν ποιούμενοι προφῆται δοκεῖν ἀντὶ τοῦ ἀστρονόμοι

The rest of the text is dedicated to rebutting the claims for the possibility of predictions and those that are carried away with this type of views, while the conclusion is a praise of reason[30]. This conclusion is interesting due to its difference to the ones of the other units. While in those we see praise to divine reason and Christian virtue, here we see emphasis on logical reasoning, which is considered the most important faculty of man.

The Σύνταγμα *by George Pachymeres*

On the other hand, Pachymeres' work is based largely on the scientific production of antiquity, it can not be characterized as a simple presentation of pre-existing knowledge. It could be an epitome of the most important achievements of ancient science, where however penetrates the perspective and the originality of a scholar who expresses the man's questions of the period. The writer does not simply copies, he selects and clarifies notions he considers difficult for his contemporaries, so as his work could fulfill its educational objectives. As a matter of fact Pachymeres considers these lessons very important, when he says that the lessons are the «completion of soul» (τελείωσις τῆς ψυχῆς τὰ μαθήματα) and he describes them as «familiar to man's mind» (οἰκεῖα τῷ ἀνθρωπίνῳ νοΐ) and congenital to it. Evenmore, he draws pleasure from the process of learning and teaching: he writes that lessons are pleasing activity (ἡ τῶν μαθημάτων θεωρία οἰκεία τε καὶ ἡδεία ἀσχολία)[31].

The differences between the *Quadrivium* of the year 1008 and *Syntagma* of Pachymeres start from the structure of the text itself. Pachymeres writes a preamble where he presents his views on the study of sciences. With references to Plato' s *Politeia*[32], he explains

βιάζονται, προφητικῆς ἐπιπνοίας καὶ χάριτος ἐπιστήμης φύσιν οὐ διαστέλλοντες, *Anonymi, Logica et Quadrivium* (cit. n. 11), 120.

[30] Μήποτε τοῦτο δόξαιμεν, εἰ μὴ τὸ κάλλιστον δῶρον τῶν ἐν ἡμῖν, τὴν ἡγεμονίαν τοῦ λογικοῦ, σφαλερῶς παραιτοίμεθα, *Anonymi, Logica et Quadrivium* (cit. n. 11), 122.

[31] Tannery, Quadrivium *de Georges Pachymère* (cit. n. 10), 5, 6.

[32] Tannery, Quadrivium *de Georges Pachymère* (cit. n. 10), 6.

mainly the usefulness of sciences in everyday life (accounts, military expeditions, navigation) and also as an opportunity for elevating the spirit of people[33]. Making his own interpretation of Plato, he believes that through knowledge aiming at the satisfaction of everyday needs, human intellect can face and understand the eternal principles of the universe, transcending material data. From this point of view, the science, according to him, is the bridge that connects material beings with ideal, an ontology that refers directly to platonic *Timaeus*, since then follows the division and the evaluation of beings into lower material as well as higher and eternal, ideal and imperishable ones. He uses, as an example, the notion of the number ten, as the quantitative characteristic of the material beings and the eternal notion of the number.

After the preamble and the presentation of the value of science, the writer deals with the material to be taught. In the unit on arithmetic, Pachymeres uses as his sources Diophantus[34] but also Nicomachus of Gerasa. As a matter of fact, continuing the elaboration on Diophantus, he solves successfully the problem of quadratic equation. Pachymeres also seems to have known the use of Arabic numbers (Hindu)[35].

[33] Ὁ κατὰ τοῦτον Σωκράτης· τοῦ γὰρ προσδιαλεγομένου ἐν τῇ κατ᾽ αὐτὸν Πολιτεία, αἰτίας τινάς εὐλόγους ἐπιφέρει δοκοῦντος τοῖς μαθήμασιν, ὡς εὔχρηστά εἰσι πρὸς τὸν ἀνθρώπινον βίον, ἡ μὲν ἀριθμητικὴ πρὸς λογισμοὺς καὶ διανομὰς καὶ συνεισφορὰς καὶ ἀμείψεις καὶ κοινωνίας, ἡ δὲ γεωμετρία πρὸς στρατοπεδεύσεις πόλεών τε καὶ ἱερῶν συγκτίσεις καὶ γεωμορίας, μουσικὴ δὲ πρὸς ἑορτὰς καὶ θυμηδίας καὶ θεῶν θρησκείας, καὶ σφαιρικὴ δὲ καὶ ἀστρονομία πρὸς γεωργίας τε καὶ ναυτιλίαν καὶ τὰς ἄλλας καταρχὰς τῶν πράξεων εὐχερείας καὶ ἐπιτηδειότητας προδηλούσα, ἐπιπλήττων φησὶ Σωκράτης· ὡς ἡδὺς εἶ ὅτι ἔοικας δεδιέναι μὴ ἄρα ἄχρηστα ταῦτα τὰ μαθήματα προστάττοιμι· τὸ δὲ ἔστι παγχάλεπον, μᾶλλον δὲ ἀδύνατον, ὄμμα γὰρ τῆς ψυχῆς ὑπὸ τῶν ἄλλων ἐπιτηδευμάτων ἀποτυφλούμενον καὶ κατορυττόμενον διὰ τούτων μόνον ἀναζωπυρεῖται καὶ ἀνεγείρεται, κρεῖττον ὃν σωθῆναι μυρίων σωματικὸν ὀμμάτων· μόνῳ γὰρ αὐτῇ ἡ περὶ τοῦ παντὸς ἀλήθεια ὁρᾶται, Tannery, *Quadrivium de Georges Pachymère* (cit. n. 10), 5, 6.

[34] George Pachymeres had paraphrased the 1st Book of *Arithmetica* by Diophantus. Cfr. P. Tannery, *Diophanti Alexandrini Opera Omnia*, I, Leipzig 1893, 78-122, with Tannery, *Quadrivium de Georges Pachymère* (cit. n. 10), ch. 44.

[35] See P. Tannery, *Les chiffres arabes dans le manuscrits grecs*, «Mémoires», 4 (1920), 199-205.

In the unit on geometry, he makes an essential reference to the work of Euclid, while in the unit on astronomy he refers to a great series of ancient astronomers, like Aratus, Archimedes, Aristotle, Cleomedes, Euclid, Claudius Ptolemaeus and Theon of Alexandria.

The work is rather extended. It follows the traditional teaching method of presentation of the general principles of every scientific field and then proceeds to further analysis, but also a presentation and solution of problems. The way of presentation and every unit is analyzed in detail and in depth. On this level the work can cover teaching by itself, that's without the scholars that constitute its source.

COMPARISON OF TWO TEXTS

The differences between the two works are numerous and important ones. Their comparative examination reveals both the level of scientific achievements of each period and the general perception regarding sciences and their legitimization. The first obvious difference with the Εὐσύνοπτον σύνταγμα of 1008 has to do with the general perception of sciences, as it appears on the level of discourse.

The unknown writer of 1008 takes care at any time to refer to the relationship between faith and science, placing the former in priority, although somewhat embarrassed, probably fearing that he may be accused as a supporter of paganism. On the other hand, Pachymeres, a teacher and official of the church, does not have such dilemmas. As a matter of fact, in the preamble he mentions that science guards man from folly and that knowledge brings man closer to God. In his work he limits himself in mentioning his general view on sciences only as an introduction, adopting the Platonic concepts of the ideal, while in the main body of the material to be taught there are no references to issues of faith. It is very interesting his reference to Platonic ontology, with no reference to Christian principles. Compared with the writer of 1008, who continuously feels the need to justify his involvement with sciences, and at the

same time to emphasize Christian faith, Pachymeres does not seem to be concerned with something similar. Here, it is clear that science leads to eternal truth.

On a content level, it is obvious that the level of Pachymeres' work is a lot higher than the previous one, both from qualitative and quantitative point of view: not only the teaching material is larger but there are many more exercises to be solved.

Our two handbooks seem to be representative of the two eras and the social function of everyone. In the centuries between the writing of these two handbooks, some ancient works were discovered, like the one by Diophantus, while others, preserved and well-known, were object of a further elaboration.

A more important scientific conclusion has to do with the general perception of secular sciences and their social function. Analysing Pachymeres' Σύνταγμα, we witness a shift relating at least to the stance towards sciences, if not towards its content: the involvement with sciences seems, in the period around 1300, to be legitimate and the dominant view is not to face them with suspicion, as related to paganism and idolatry. This attitude towards sciences, that is generally towards nature and the transcendental realm, will contribute significantly to the scientific discussion that will take place throughout the 14th century. The same topic is stimulating for the modern historian of the Medieval history of Science.